Convection Heat Transfer

Textbook: ***Convection Heat Transfer***

Adrian Bejan, John Wiley & Sons

Reference: ***Convective Heat and Mass Transfer***

Kays, Crawford, and Weigand, McGraw-Hill

Convection Heat Transfer

Vedat S. Arpaci and Poul S. Larsen, Prentice-Hall Inc

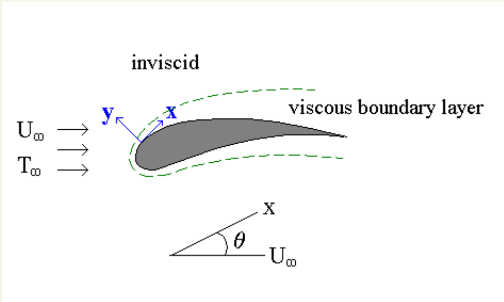
Convection Heat Transfer

Content:

1. Fluid Properties and Conservation Laws
2. External/Internal Laminar Flows
3. External/Internal Natural Convection
4. External/Internal Turbulent Flows
5. High Speed Flows

Grading: HW (30%)+Midterm (30%) + Final (30%) + Report (15%)

Wanted



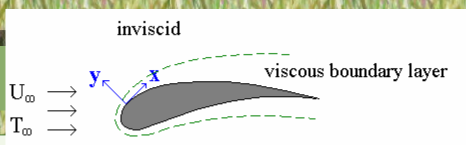
• friction force: $F_f = \oint_S \tau_0 \cos\theta \, dA$

• pressure drag: $F_p = \oint_S P_0 \sin\theta \, dA$

Skin friction coefficient $C_f \equiv \dfrac{\tau_0}{\frac{1}{2}\rho U_\infty^2}$

form drag coefficient $C_p \equiv \dfrac{F_p}{\frac{1}{2}\rho U_\infty^2 A}$

Wanted



• heat transfer rate at the surface

no-slip hypothesis at wall : pure conduction adjacent to the wall

Fourier's law: $q_0'' = -k\left(\dfrac{\partial T}{\partial y}\right)_{y=0}$

• convection heat transfer coefficient: $q_0'' \equiv h(T_0 - T_\infty)$ i.e. $h \equiv \dfrac{-k\left(\dfrac{\partial T}{\partial y}\right)_{y=0}}{(T_0 - T_\infty)}$

• local Nusselt number: $Nu_x \equiv \dfrac{hx}{k} \sim \dfrac{\text{heat transfer rate when in flow}}{\text{heat transfer rate when stationary}}$

Wanted

• averaged convection heat transfer coefficient:

$$h_{0-x} \equiv \frac{q''_{0-x}}{\Delta T_{avg}} = \frac{\frac{1}{x}\int_0^x q''_0 dx}{\Delta T_{avg}}$$

total heat transfer rate over (0,*x*) = $\int_0^x q''_0 dx = x \cdot h_{0-x} \cdot \Delta T_{avg}$

• overall Nusselt number: $Nu_{0-x} \equiv \frac{h_{0-x} x}{k} = \frac{q''_{0-x}}{\Delta T_{avg}} \frac{x}{k}$

Analysis Methods

• scaling analysis : qualitative analysis

magnitude of order

related to what parameters and how?

• integral analysis : quantitative analysis

magnitudes with a little errors

related to what parameters and how?

• similarity analysis : exact analysis

under model assumptions

• perturbation analysis : critical analysis

near some critical point

Fluid Properties

(1) viscosity coefficient: $\mu \ (\mathrm{kg/m \cdot sec})$; $\nu \equiv \mu/\rho \ (\mathrm{m^2/sec})$

- Newtonian Fluids:

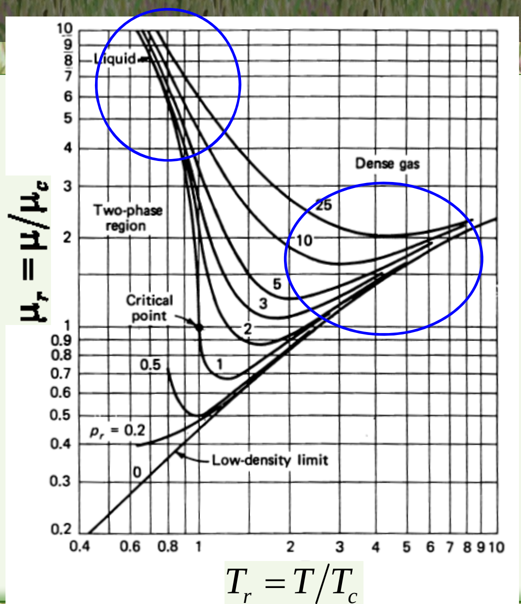
$$\sigma_{ji} = -p\delta_{ji} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \lambda\Theta\delta_{ji} \quad \equiv \ -p\delta_{ji} + \tau_{ji}$$

$$\Theta \equiv \nabla \cdot \vec{u} = \frac{\partial u_k}{\partial x_k}$$

$\lambda = -\frac{2}{3}\mu$ usually assumed (*Fluid Mechanics*, Landau & Lifschifz, 1959)

- temperature dependence:

$$\mu = \mu(T, P)$$



gases : $\mu \uparrow$ as $T \uparrow$

liquid : $\mu \downarrow$ as $T \uparrow$

Fluid Properties

(2) thermal conductivity: $k \; (\mathrm{W/m \cdot K})$; $\alpha \equiv k/\rho c_p \; (\mathrm{m^2/sec})$

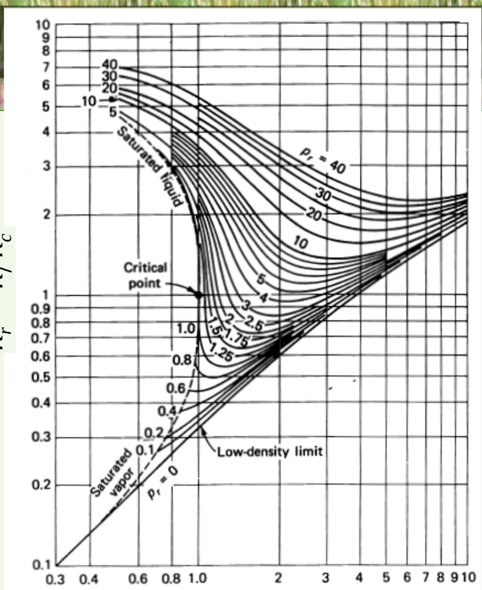
- Fourier's Law: $\vec{q}''$ (heat flux, $\mathrm{W/m^2}$) $= -k\nabla T$

  isotropic k usually assumed

- temperature dependence:

  $k = k(T, P)$

$k_r = k/k_c$



$T_r = T/T_c$

gases : $k \uparrow$ as $T \uparrow$

liquid : $k \downarrow$ as $T \uparrow$

Dimensionless Parameters

(1) Reynolds number: $\mathrm{Re} \equiv \dfrac{UL}{\nu} \sim \dfrac{\text{inertial force}}{\text{viscous force}}$

$$\mathrm{Re} = \frac{L^2/\nu}{L/U} \sim \frac{\text{char. diffusion time}}{\text{char. convection time}}$$

$$\mathrm{Re} > \mathrm{Re}_{cr} \quad \Rightarrow \quad \text{turbulent flows}$$

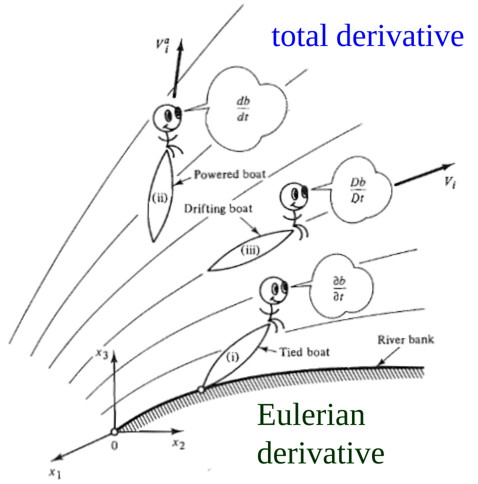
(2) Prandtl number: $\mathrm{Pr} \equiv \dfrac{\nu}{\alpha} \sim \dfrac{\text{momentum diffusion}}{\text{thermal diffusion}}$

(3) Eckert number: $Ec \equiv \dfrac{U^2}{c_p \Delta T} \sim \dfrac{\text{kinetic energy per unit mass}}{\text{enthalpy difference per unit mass}}$

$Ec << 1 \Rightarrow$ negligible viscous dissipation

High speed flows $\Rightarrow$ significant viscous dissipation

Time Derivatives



Three time derivatives

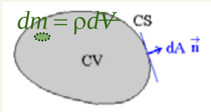
total derivative $\dfrac{d}{dt} = \dfrac{\partial}{\partial t} + \vec{V}_{observer} \cdot \nabla$

Lagrangian/ material derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{u} \cdot \nabla$$

Eulerian derivative

Conservation Laws



Let ϕ = some physical quantity per unit mass

total amount of quantity ϕ within the control volume (CV) = $\int_{CV} \phi \rho dV$

outflow rate of quantity ϕ through the control surface (CS) = $\oint_{CS} \phi \rho \vec{u} \cdot \vec{n} dA$

Conservation requires : $\frac{\partial}{\partial t} \int_{CV} \phi \rho dV + \oint_{CS} \phi \rho \vec{u} \cdot \vec{n} dA$ = sources = $\int_{CV} \dot{q} dV$

$\dot{q}$ = source per unit time per unit volume

Differential Form

Divergence Theorem: $\oint_S \vec{a} \cdot \vec{n} dA = \int_V \nabla \cdot \vec{a}\, dV = \int_V \frac{\partial a_j}{\partial x_j} dV$

Conservation requires: $\frac{\partial}{\partial t} \int_{CV} \phi \rho dV + \oint_{CS} \phi\, \rho\, \underline{\vec{u}} \cdot \vec{n} dA$ = sources = $\int_{CV} \dot{q} dV$

Conservation law: $\frac{\partial}{\partial t} \int_{CV} \phi \rho dV + \int_{CV} \nabla \cdot (\phi\, \rho\, \vec{u}) dV$ = sources = $\int_{CV} \dot{q} dV$

- Consider an infinitesimal control volume $CV = dV$

$$\frac{\partial}{\partial t} \phi \rho dV + \nabla \cdot (\phi\, \rho\, \vec{u}) dV = \dot{q} dV$$

$$\boxed{\frac{\partial}{\partial t}(\phi \rho) + \nabla \cdot (\phi\, \rho\, \vec{u}) = \dot{q}}$$

Mass Conservation

total amount of quantity ϕ within the control volume (CV) $= \int_{CV} \phi \rho \, d\forall$

$$\frac{\partial}{\partial t}(\phi \rho) + \nabla \cdot (\phi \rho \vec{u}) = \dot{q}$$

• mass: $\phi = 1$, no source $\dot{q} = 0$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

vector identity:

$$\nabla \cdot (\varphi \vec{a}) = \vec{a} \cdot \nabla \varphi + \varphi \nabla \cdot \vec{a}$$

$$\frac{\partial \rho}{\partial t} + \vec{u} \cdot \nabla \rho + \rho \nabla \cdot \vec{u} = \frac{D\rho}{Dt} + \rho \nabla \cdot \vec{u} = 0$$

$$\nabla \cdot \vec{u} = -\frac{1}{\rho}\frac{D\rho}{Dt} = \frac{1}{\upsilon}\frac{D\upsilon}{Dt} =$$ volume change rate per unit volume

Momentum Conservation

total amount of quantity ϕ within the control volume (CV) $= \int_{CV} \phi \rho \, d\forall$

• momentum $\phi = u_i$

Divergence Theorem: $\oint_S \vec{a} \cdot \vec{n} dA = \int_V \nabla \cdot \vec{a} \, d\forall = \int_V \frac{\partial a_j}{\partial x_j} d\forall$

source $\int_{CV} \dot{q} d\forall =$ force acting on the CV by its surrounding fluids

$=$ body forces + contact forces

$$= \int_{CV} X_i d\forall + \oint_{CS} \sigma_{ji} n_j dA = \int_{CV} X_i d\forall + \int_{CV} \frac{\partial \sigma_{ji}}{\partial x_j} d\forall$$

$$\dot{q} = X_i + \frac{\partial \sigma_{ji}}{\partial x_j}$$

Momentum Conservation

$$\frac{\partial}{\partial t}(\phi\rho)+\nabla\cdot(\phi\rho\vec{u})=\dot{q}$$

• momentum $\phi = u_i$

$$\dot{q}=X_i+\frac{\partial\sigma_{ji}}{\partial x_j}$$

$$\frac{\partial}{\partial t}(\rho u_i)+\nabla\cdot(\rho u_i\vec{u})=X_i+\frac{\partial\sigma_{ji}}{\partial x_j}$$

$$u_i\left\{\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec{u})=0\right\}$$

Newtonian fluid: $\sigma_{ji}=-p\delta_{ji}+\tau_{ji}$

$$\frac{\partial\sigma_{ji}}{\partial x_j}=\frac{\partial}{\partial x_j}\left(-p\delta_{ji}+\tau_{ji}\right)=-\delta_{ji}\frac{\partial p}{\partial x_j}+\frac{\partial\tau_{ji}}{\partial x_j}=-\frac{\partial p}{\partial x_i}+\frac{\partial\tau_{ji}}{\partial x_j}$$

Newtonian fluid: $\rho\frac{\partial u_i}{\partial t}+(\rho\vec{u}\cdot\nabla)u_i = X_i-\frac{\partial p}{\partial x_i}+\frac{\partial\tau_{ji}}{\partial x_j}$

Energy Conservation

• total energy: $\phi = e$ (internal energy per unit mass) $+\frac{1}{2}u_iu_i+V$ (potential energy)

body force $\vec{X}=-\rho\nabla V$

• sources $=$ exteranl hea[t]

heat diffusio[n]

Divergence Theorem: $\oint_S \vec{a}\cdot\vec{n}dA=\int_V\nabla\cdot\vec{a}\,d\forall=\int_V\frac{\partial a_j}{\partial x_j}d\forall$

work done on the CV by its surroundin gs

$a_j=\sigma_{ji}u_i$

$$=\int_{CV}\dot{q}d\forall \quad - \quad \oint_{CS}\vec{q}''\cdot\vec{n}dA \quad + \quad \oint_{CS}(\sigma_{ji}n_jdA)\,u_i$$

$$\Rightarrow \frac{\partial}{\partial t}(\phi\rho)+\nabla\cdot(\phi\rho\vec{u})=\dot{q}-\nabla\cdot\vec{q}''+\frac{\partial}{\partial x_j}(\sigma_{ji}u_i) \longleftarrow \sigma_{ji}=-p\delta_{ij}+\tau_{ji}$$

Energy Conservation

- total energy equation:

$$-p\frac{\partial u_i}{\partial x_i} - u_i\frac{\partial p}{\partial x_i} + u_i\frac{\partial \tau_{ji}}{\partial x_j} + \tau_{ji}\frac{\partial u_i}{\partial x_j}$$

$$\rho\frac{D}{Dt}\left(e + \tfrac{1}{2}u_i u_i + V\right) = \dot{q} - \nabla\cdot\vec{q}'' - \frac{\partial}{\partial x_i}(p u_i) + \frac{\partial}{\partial x_j}(u_i \tau_{ji})$$

- kinetic energy equation: $u_i \cdot \left\{\rho\frac{Du_i}{Dt} = X_i - \frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ji}}{\partial x_j}\right\}$ $\qquad \vec{X} = -\rho\nabla V$

$$\rho\frac{DV}{Dt} = \rho\left(\frac{\partial V}{\partial t} + u_i\frac{\partial V}{\partial x_i}\right) = 0 - u_i X_i = -u_i X_i$$

- thermal energy equation:

$$\rho\frac{De}{Dt} = \dot{q} - \nabla\cdot\vec{q}'' - p\nabla\cdot\vec{u} + \mu\Phi$$

$$\mu\Phi \equiv \tau_{ji}\frac{\partial u_i}{\partial x_j}$$

thermal (internal) energy

$$\rho\frac{De}{Dt} = \dot{q} - \nabla\cdot\vec{q}'' - p\nabla\cdot\vec{u} + \mu\Phi$$

the time change rate of the internal energy of an infinitesimal control volume = the heat generation rate

+ the net heat diffusion rate

+ pressure work rate done by surrounding fluid

+ viscous dissipation rate

Thermal Energy Conservation

$$\rho \frac{De}{Dt} = \dot{q} - \nabla \cdot \vec{q}'' - p\nabla \cdot \vec{u} + \mu\Phi$$

$\tau_{ij} \dfrac{\partial u_i}{\partial x_j} \equiv \mu\Phi$ = viscous dissipation rate = rate at which kinetic energy is irreversibly converted to thermal energy by viscosity

Newtonian fluid: $\tau_{ij} = \mu\left(\dfrac{\partial u_i}{\partial x_j} + \dfrac{\partial u_j}{\partial x_i}\right) + \lambda\Theta\delta_{ij}$

$$\Rightarrow \quad \mu\Phi = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j} + \lambda\Theta\delta_{ij}\frac{\partial u_i}{\partial x_j}$$

$$= \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\left\{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)\right\} + \lambda\Theta^2$$

$$= \frac{\mu}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \lambda\Theta^2$$

$\tau_{ij} \dfrac{\partial u_i}{\partial x_j} \equiv \mu\Phi$ = **viscous dissipation rate**

$$\mu\Phi = \frac{\mu}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \lambda\Theta^2 \quad = \frac{\mu}{2}\left\{\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{4}{3}\frac{\partial u_i}{\partial x_i}\frac{\partial u_j}{\partial x_j}\right\}$$

$$= \frac{\mu}{2}\left\{\frac{8}{3}\left(\frac{\partial u_1}{\partial x_1}\right)^2 + \frac{8}{3}\left(\frac{\partial u_2}{\partial x_2}\right)^2 + \frac{8}{3}\left(\frac{\partial u_3}{\partial x_3}\right)^2 + \sum_{\substack{i,j=1\\ i\neq j}}^{3}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2 - \frac{8}{3}\left(\frac{\partial u_1}{\partial x_1}\frac{\partial u_2}{\partial x_2} + \frac{\partial u_1}{\partial x_1}\frac{\partial u_3}{\partial x_3} + \frac{\partial u_2}{\partial x_2}\frac{\partial u_3}{\partial x_3}\right)\right\}$$

$$= \frac{\mu}{2}\left\{\begin{array}{l}\frac{4}{3}\left[\left(\frac{\partial u_1}{\partial x_1}\right)^2 - 2\frac{\partial u_1}{\partial x_1}\frac{\partial u_2}{\partial x_2} + \left(\frac{\partial u_2}{\partial x_2}\right)^2\right] + \frac{4}{3}\left[\left(\frac{\partial u_1}{\partial x_1}\right)^2 - 2\frac{\partial u_1}{\partial x_1}\frac{\partial u_3}{\partial x_3} + \left(\frac{\partial u_3}{\partial x_3}\right)^2\right] \\ + \frac{4}{3}\left[\left(\frac{\partial u_2}{\partial x_2}\right)^2 - 2\frac{\partial u_2}{\partial x_2}\frac{\partial u_3}{\partial x_3} + \left(\frac{\partial u_3}{\partial x_3}\right)^2\right] + \sum_{\substack{i,j=1\\ i\neq j}}^{3}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2\end{array}\right\}$$

$$= \frac{\mu}{2}\left\{\frac{4}{3}\left[\left(\frac{\partial u_1}{\partial x_1} - \frac{\partial u_2}{\partial x_2}\right)^2 + \left(\frac{\partial u_1}{\partial x_1} - \frac{\partial u_3}{\partial x_3}\right)^2 + \left(\frac{\partial u_3}{\partial x_3} - \frac{\partial u_2}{\partial x_2}\right)^2\right] + \sum_{\substack{i,j=1\\ i\neq j}}^{3}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2\right\} \quad \mathbf{> 0}$$

Temperature-based

$$\rho\frac{De}{Dt} = \dot{q} - \nabla\cdot\vec{q}'' - p\nabla\cdot\vec{u} + \mu\Phi$$

$h = e + p\upsilon$

$$dh = de + dp\upsilon = Tds - pd\upsilon + dp\upsilon = Tds + \upsilon dp$$

(first law)

$s(T, p)$

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp \quad = \left(\frac{\partial s}{\partial T}\right)_p dT - \frac{\beta}{\rho}dp$$

$$\frac{\partial(s,T)}{\partial(\upsilon,p)} = 1 \qquad \left(\frac{\partial s}{\partial p}\right)_T = \frac{\partial(s,T)}{\partial(p,T)} = \frac{\partial(\upsilon,p)}{\partial(p,T)} = -\frac{\partial(p,\upsilon)}{\partial(p,T)} = -\left(\frac{\partial \upsilon}{\partial T}\right)_p = -\beta\upsilon = -\frac{\beta}{\rho}$$

(Maxwell relation)

$$dh = T\left(\frac{\partial s}{\partial T}\right)_p dT + \left(\upsilon - T\frac{\beta}{\rho}\right)dp$$

Temperature-based

$$\rho\frac{De}{Dt} = \dot{q} - \nabla\cdot\vec{q}'' - p\nabla\cdot\vec{u} + \mu\Phi$$

$$dh = T\left(\frac{\partial s}{\partial T}\right)_p dT + \left(\upsilon - T\frac{\beta}{\rho}\right)dp \quad \Rightarrow \quad h = h(T, p)$$

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p = T\left(\frac{\partial s}{\partial T}\right)_p$$

$$\Rightarrow \quad dh = c_p dT + (1 - \beta T)\frac{dp}{\rho}$$

$$\rho\frac{Dh}{Dt} = \rho c_p\frac{DT}{Dt} + (1 - \beta T)\frac{Dp}{Dt}$$

Temperature-based

• enthaphy $h = e + p/\rho \quad \Rightarrow \quad \frac{Dh}{Dt} = \frac{De}{Dt} + \frac{1}{\rho}\frac{Dp}{Dt} - \frac{p}{\rho}\cdot\frac{1}{\rho}\frac{D\rho}{Dt} = \frac{De}{Dt} + \frac{1}{\rho}\frac{Dp}{Dt} + \frac{p}{\rho}\nabla\cdot\vec{u}$

$$\rho\frac{De}{Dt} = \dot{q} - \nabla\cdot\vec{q}'' - p\nabla\cdot\vec{u} + \mu\Phi$$

$$\rho\frac{Dh}{Dt} = \rho c_p\frac{DT}{Dt} + (1-\beta T)\frac{Dp}{Dt}$$

$$\rho c_p\frac{DT}{Dt} + (1-\beta T)\frac{Dp}{Dt} = \dot{q} - \nabla\cdot\vec{q}'' - p\nabla\cdot\vec{u} + \mu\Phi + \frac{Dp}{Dt} + p\nabla\cdot\vec{u}$$

$$\Rightarrow \quad \rho c_p\frac{DT}{Dt} = \dot{q} - \nabla\cdot\vec{q}'' + \beta T\frac{Dp}{Dt} + \mu\Phi$$

Temperature

$$\rho c_p\frac{DT}{Dt} = \dot{q} - \nabla\cdot\vec{q}'' + \beta T\frac{Dp}{Dt} + \mu\Phi$$

$$\beta \equiv -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p = \frac{1}{\upsilon}\left(\frac{\partial\upsilon}{\partial T}\right)_p \quad = \text{thermal expansion coefficient}$$

• Assumptions: (1) negligible compressibility effect $\beta = 0$

(2) no external heat generation $\dot{q} = 0$

(3) negligible viscous dissipation $\mu\Phi \approx 0 \quad (Ec << 1)$

(4) Fourier's Law: $\vec{q}'' = -k\nabla T$

$$\rho c_p\frac{DT}{Dt} = \rho c_p\left(\frac{\partial T}{\partial t} + \vec{u}\cdot\nabla T\right) = \nabla\cdot(k\nabla T)$$