Course name: ***Introduction to Turbulence***

Instructor: 黃美嬌

Reference books:

1. **An Introduction to Turbulent Flow/Jean Mathieu & Julian Scott, Cambridge University Press, 2000**
2. **Turbulent Flows/S.B. Pope (Cambridge)**
3. The physics of fluid turbulence/W.D. McComb
4. The theory of homogeneous turbulence/G.K. Batchelor
5. A first course in turbulence/H. Tennekes & J.L. Lumle

Content:

1. Regularity, Chaos, and Turbulence
2. Characteristics of Turbulence
3. Vorticity and Vortex Stretching
4. Statistical Tools and Spectral Analysis
5. Space and Time Scales of Turbulence
6. Mean Motion
7. Classical Models of Jets, Wakes, and Boundary Layers
8. Spectral Analysis of Homogeneous Turbulence
9. Kolmogorov's and other Spectral Theories
10. DNS, LES, and RANS

1. Regularity, Chaos, and Turbulence

1. Regularity, Chaos, and Turbulence

- ABC Flows (Arnold(1965)+Beltrami+Childress(1970)

$$u = A\sin z + C\cos y$$
$$v = B\sin x + A\cos z$$
$$w = C\sin y + B\cos x$$
$$C \le B \le A = 1$$

(1) laminar flows (Eulerian viewpoint)

(2) a solution of steady incompressible Euler equations

$$\nabla\cdot\vec{u} = 0$$
$$\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = -\nabla\frac{p}{\rho}$$

Ref: Dombre et al, JFM 167, 353-391 (1986)

1. Regularity, Chaos, and Turbulence

(3) Possess Beltrami properties: $\nabla \times \vec{u} = \vec{\omega} = \lambda \vec{u}$; $\vec{u} \cdot \nabla \lambda = 0$

(4) Integrable if C = 0

$$u = A \sin z + C \cos y$$
$$v = B \sin x + A \cos z$$
$$w = C \sin y + B \cos x$$
$$C \le B \le A = 1$$

pathlines:

$$u = \frac{dx}{dt} = A \sin z$$

$$v = \frac{dy}{dt} = B \sin x + A \cos z$$

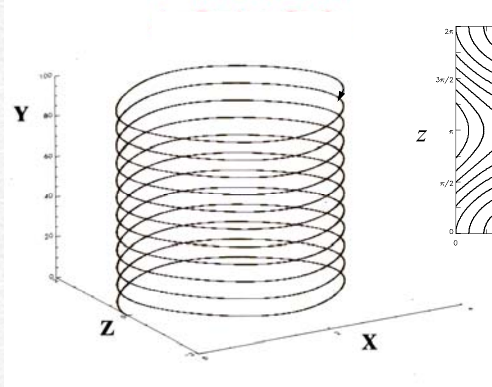
$$w = \frac{dz}{dt} = B \cos x$$

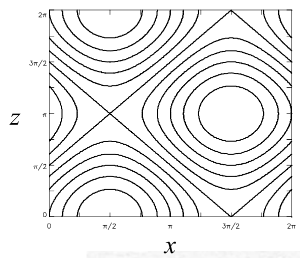
$$\Rightarrow \ B \cos x \cdot \frac{dx}{dt} - A \sin z \cdot \frac{dz}{dt} = 0$$

$$\Rightarrow \ B \sin x + A \cos z = \text{constant} = V \ ; \quad y(t) = y(0) + Vt$$

1. Regularity, Chaos, and Turbulence

• Pathlines: A=1,B=1,C=0 (regular) $\Rightarrow$ $\sin x + \cos z =$ constant

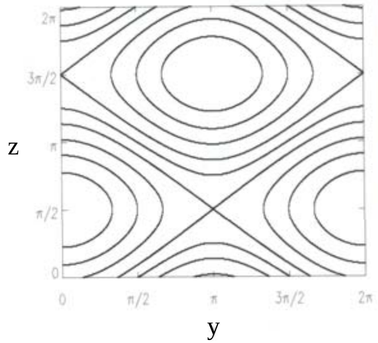




1. Regularity, Chaos, and Turbulence

• contours of constant magnitudes: A=1,B = C = $1/\sqrt{2}$

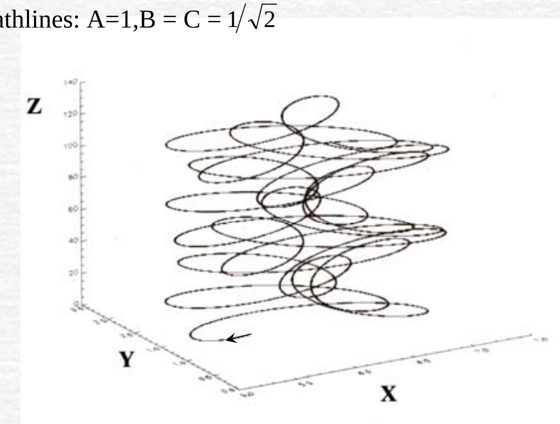
(Eulerian viewpoint: regular)



$$u = A \sin z + C \cos y$$
$$v = B \sin x + A \cos z$$
$$w = C \sin y + B \cos x$$
$$C \le B \le A = 1$$

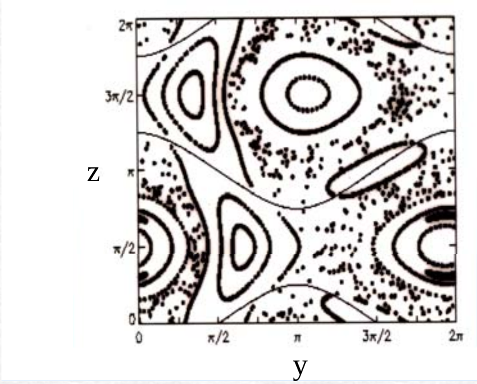
1. Regularity, Chaos, and Turbulence

• Pathlines: A=1,B = C = $1/\sqrt{2}$



(Lagrangian turbulence)

- Poincare maps



- Duffing Equation $\frac{d^2x}{dt^2} + \delta \frac{dx}{dt} - x + x^3 = \gamma \cos \omega t$

  δ: damping coefficient

  γ: forcing amplitude

  ω: forcing frequency

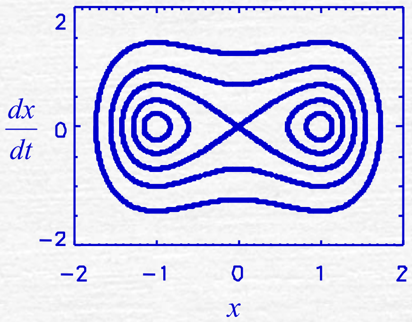
$if \ \ \delta = \gamma = 0$, then integrable

$if \ \gamma = 0$,

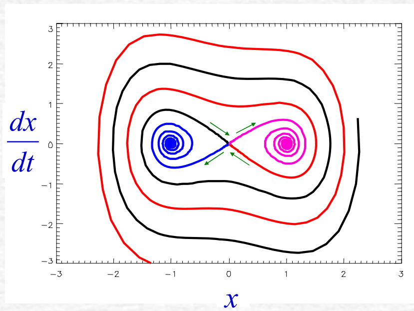
$0 < \delta < 2\sqrt{2}$: stable spiral nodes at $(x = \pm 1, \frac{dx}{dt} = 0)$

$\delta \geq 2\sqrt{2}$: stable nodes at $(x = \pm 1, \frac{dx}{dt} = 0)$

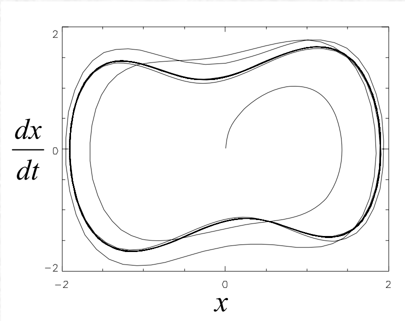
- phase space $(\delta = \gamma = 0)$:



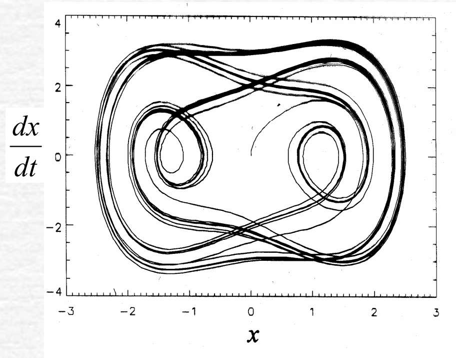
- phase space $(\gamma = 0, \ \delta < \ 2\sqrt{2})$:



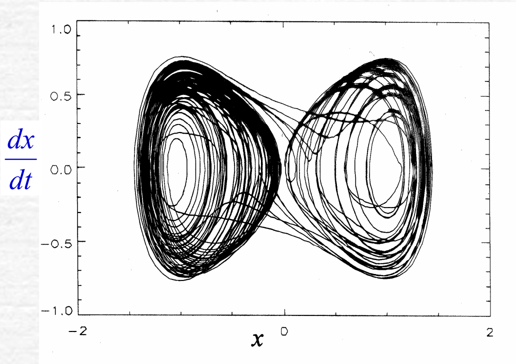
• phase space ($\gamma = 1.2$, $\delta = 0.5$, $\omega = 1$) :



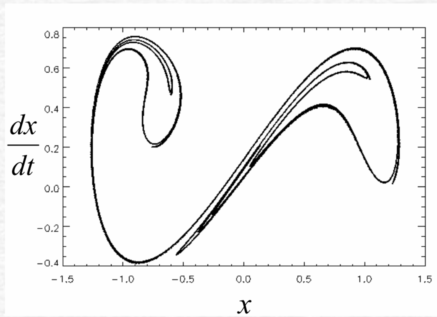
• phase space ($\gamma = 1.6$, $\delta = 0.1$, $\omega = 1$) :



• phase space ($\gamma = 0.32$, $\delta = 0.32$, $\omega = 1.2$) :



• Poincare map ($\gamma = 0.32$, $\delta = 0.32$, $\omega = 1.2$) :

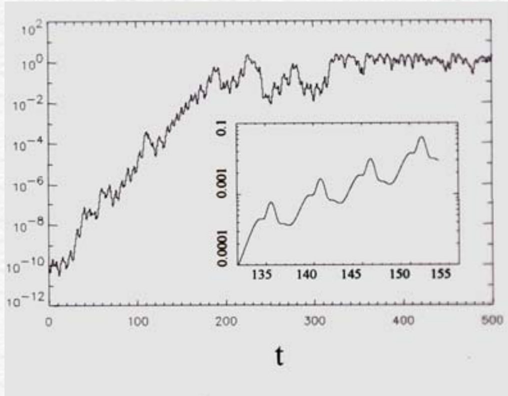


Chaotic Systems:

- dynamical systems with a limited number of degrees of freedom
- chaotic in time in the sense that two initially close-by points in the phase space will separate exponentially

Turbulent Flows:

- have infinite number of degrees of freedom
- chaotic both in time and in space



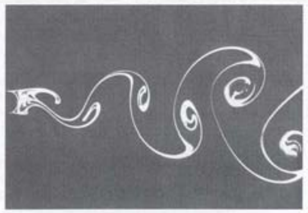
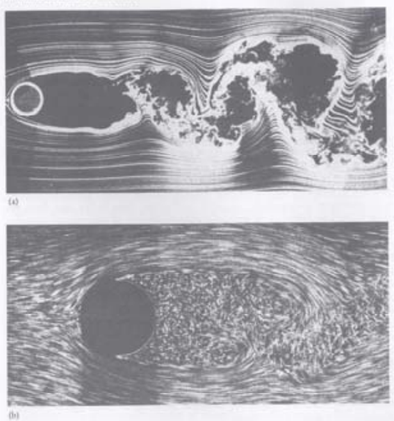


Figure 1.1. The Von Karman vortex street behind a cylinder placed in a uniform flow. (Courtesy of Sadotoshi Taneda.)

laminar v.s. turbulent



(a)

(b)

Figure 1.2. Visualizations of a turbulent cylinder wake; (b) shows a close-up. ((a) Courtesy of Thomas Corke and Hassan Nagib; (b) ONERA photograph, Werlé and Gallon (1972), reproduced with permission.)

2. Characteristics of Turbulence

(1) Unpredictable

~ any small disturbance causes dramatically different results

~ ir-reproducible

~ by nature unstable in the sense that a small disturbance will be amplified by nonlinearity

c.f. **laminar – disturbance is damped by viscosity**

(2) Random fluctuations in all physical quantities

~ fluctuations both in time and in space

~ statistical tools may be helpful;

< u >, $3q^2/2$=mean fluctuation energy (turbulent intensity)

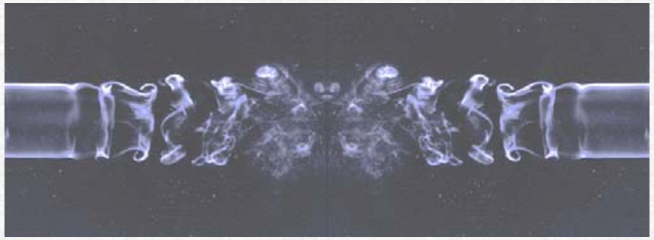
~ follows Navier-Stokes equations (continuum)

(3) Involve a wide range of length and time scales

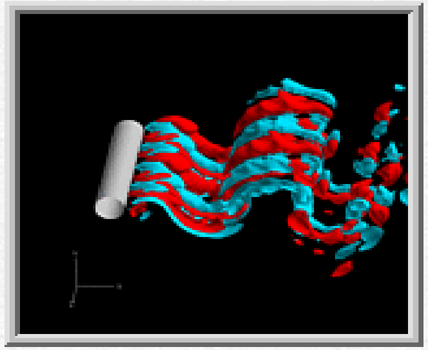
cf. laminar – one or several dominant frequencies/wavelengths

❖ large scales (L)

- fixed by the overall geometry of flow
- contain most of turbulent (kinetic) energy ($3q^2/2$)
- anisotropic and nonhomogeneous
- depends on boundaries, external forces, etc.



double jets: L = jet width



L = cylinder diameter

Re = 300 (Arthur Kravchenko)

2. Characteristics of Turbulence

❖ small scales (Kolmogorov's dissipation length, η)

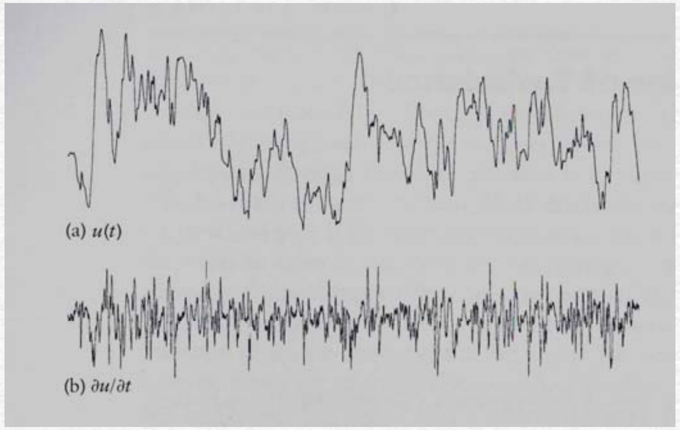
- dissipate most of the energy
- flow adjust itself according to viscosity
- approximately isotropic, homogeneous, universal
- weakly affected by boundaries, external forces, etc.
- important parameters:

ε = energy dissipation rate per unit mass $(m^2/\sec^3)$

ν = momentum diffusivity $(m^2/\sec)$

$$\eta = \left(\nu^3/\varepsilon\right)^{1/4} \quad ; \quad \tau_\eta = \left(\nu/\varepsilon\right)^{1/2}$$

2. Characteristics of Turbulence



2. Characteristics of Turbulence

❖ Range of length/time scales

dimensional analysis: $\eta = \left(\nu^3/\varepsilon\right)^{1/4} \quad ; \quad \tau_\eta = \left(\nu/\varepsilon\right)^{1/2}$

Empirical relation: $L = q^3/\varepsilon \quad ; \quad \tau_L = L/q$

Range of time/length scales:

$$L/\eta = \mathrm{Re}_L^{3/4}$$

$$\tau_L/\tau_\eta = \mathrm{Re}_L^{1/2}$$

$$\mathrm{Re}_L \equiv \frac{qL}{\nu}$$

Ex. d.o.f. in space = 10^9 for $\mathrm{Re}_L = 10^4$

2. Characteristics of Turbulence

Shear Layer Thickness $L \approx 40$ mm

Kolmogrov length scale, $\eta = L\,\mathrm{Re}_L^{-3/4} = \left(\frac{\nu^3 L}{U^3}\right)^{1/4}$

For air $(\nu = 1.5\times10^{-5}\,\mathrm{m}^2/\sec)$

if $U = 3.3\,\mathrm{m/sec}$ and $L = 40\mathrm{mm}$

$\Rightarrow \eta = 0.044\,\mathrm{mm}$

2. Characteristics of Turbulence

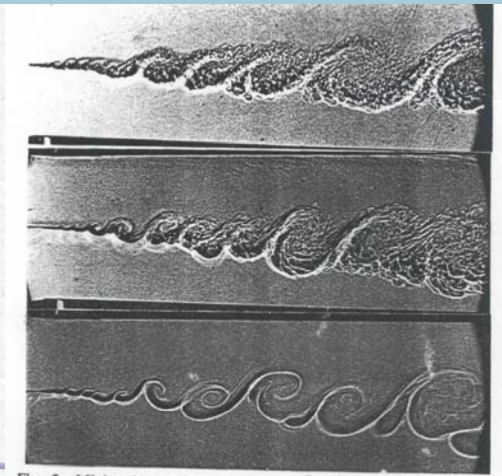


Fig. 2 Mixing layer between helium and nitrogen $U_2/U_1=0.38$; $\rho_2/\rho_1=7$; $\rho_1 L_1 L/\mu_1=1.2$, 0.6 and 0.3×10^5, respectively, from top to bottom ...

2. Characteristics of Turbulence

❖ intermediate scales

- inertial scales called usually
- exist only when Re is sufficiently large

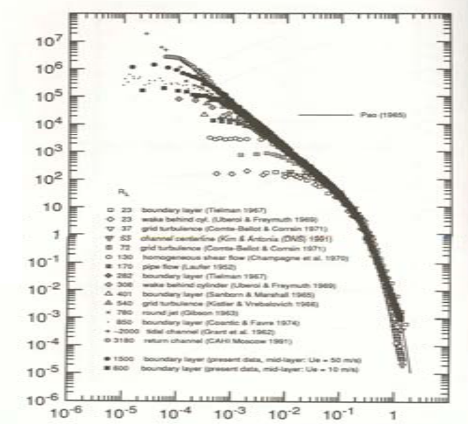
  free from large-scale effects

  free of dissipation

- energy spectrum: $E(k) = C_K \varepsilon^{2/3} k^{-5/3}$

  experimentally supported and theoretically derived

$$E_{11}(k_1)/\left(\varepsilon\nu^5\right)^{1/4}$$



$k_1\eta$

2. Characteristics of Turbulence

(4) Strong mixing ability

~ larger friction drag

~ more resistant to adverse pressure gradients

e.g. golf ball, baseball

~ powerful dispersal of material and heat

molecular diffusivities: α and ν (m^2/sec)

turbulent diffusivities: Lq (large scale*turbulent velocity)

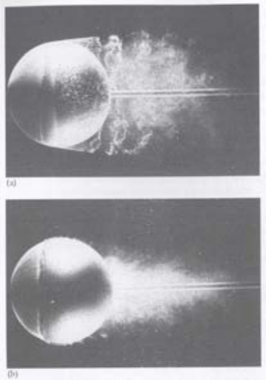


Figure 1.8. A solid sphere placed in uniform flow: (a) the boundary layer is laminar and separates to form the wake behind the sphere; (b) a trip wire has triggered boundary-layer transition upstream of where flow separation occurs in Figure 1.8a and hence delayed separation. (ONERA photograph, Werlé (1980), reproduced with permission.)

laminar

turbulent

Consider a room of size L:

Molecular dispersal time scale $T_\alpha \approx L^2/\alpha$

Turbulent dispersal time scale $T_q \approx L/q$

$$\left.\begin{array}{l} T_\alpha \approx L^2/\alpha \\ T_q \approx L/q \end{array}\right\} ratio = \frac{Lq}{\alpha}$$

air , $L = 5m$, $q = 0.05\, m/s$, $\alpha = 0.208\,\mathrm{cm}^2/\mathrm{s}$

$T_\alpha \sim 10^6 s$

$T_q \sim 10^2 s$

$ratio \sim 10^4$!

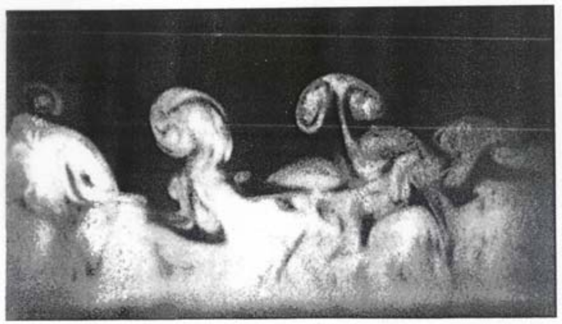
(5) Intermittency

(a) External intermittency

~ recognizable coherent structures

(some flow patterns which may have many random features but nevertheless occurs with sufficient regularity in space and/or time)

~ appears at some particular locations and times



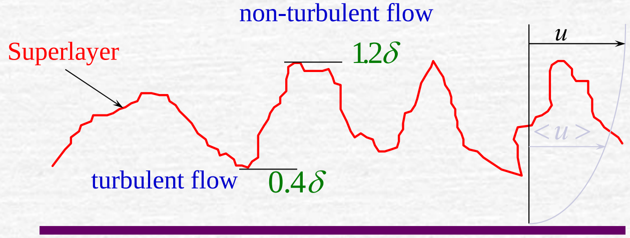
wall turbulence: mushroom structures

2. Characteristics of Turbulence

Jupiter

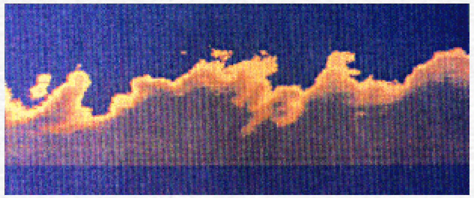
2. Characteristics of Turbulence

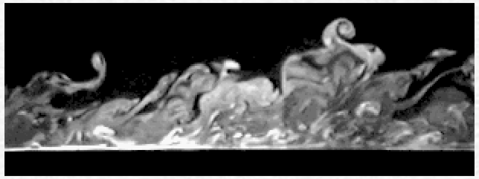
intermittent turbulent boundary layer



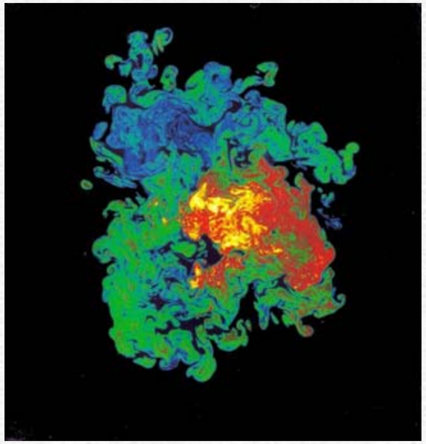
2. Characteristics of Turbulence

intermittent turbulent boundary layer





2. Characteristics of Turbulence

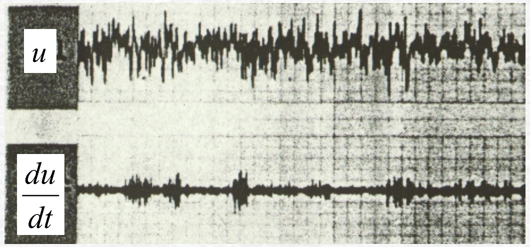


Jet (Dimotakis)

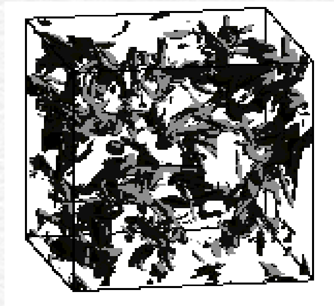
2. Characteristics of Turbulence

(b) Internal intermittency

~ intermittent distribution in time/space of small-scale quantities (such as vorticity, dissipation rate, and so on)



2. Characteristics of Turbulence



small-scale structures

energy cascade

spectral dynamics

fractal/multifractal

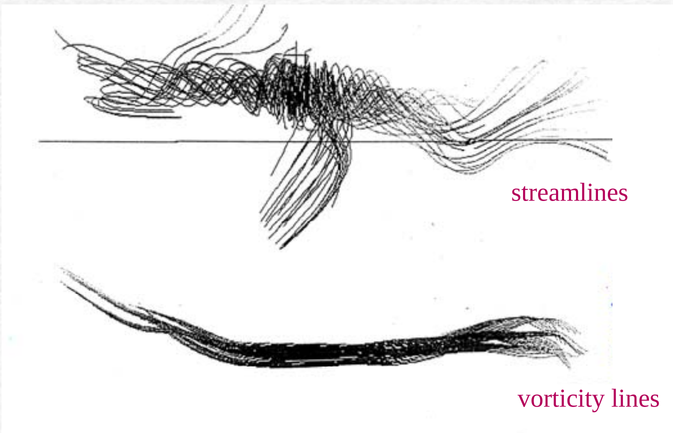
Iso-surfaces of enstrophy $Re_\lambda = 59$

pseudospectral method (120^3)

G.R. Ruetch & J.H. Ferziger (1997)

$Re_\lambda \approx 200$ for 512^3

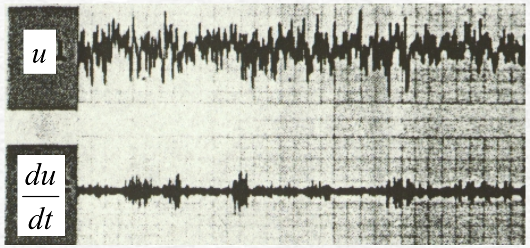
2. Characteristics of Turbulence



2. Characteristics of Turbulence

(b) Internal intermittency

~ Fractal (irregular/fragmented, e.g. Cantor set, Koch curve)



2. Characteristics of Turbulence

(b) Internal intermittency

~ Self-similar (invariant under change of scale)

2. Characteristics of Turbulence

Cantor set

$D = \log 2/\log 3 = 0.63$

Koch curve

$D = \log 4/\log 3 = 1.26$

2. Characteristics of Turbulence

(6) Self-Sustaining (Energy Cascade)

~ large eddies extract energy from mean shear

~ smaller eddies extract energy from large eddies

~ smallest eddies dissipate energy

(7) Rotational

~ $\vec{\omega} = \nabla \times \vec{u} \neq 0$ at least for certain regions of space

~ sources: viscosity+solid wall, pressure+stratification, gravity

~ growth: vortex stretching

~ decay: viscosity diffusion

Some Questions

1. Why turbulence occur?
2. Why large eddies are energy-containing eddies?
3. Why small eddies take charge of dissipation?
4. How is the energy cascaded?
5. Why is the energy so cascaded?
6. How to ***predict*** (analytically and/or numerically) turbulence qualitatively and/or quantitatively?